Reducing the Influence of DC Offset Drift in analog IPs using the Thue-Morse Sequence as Stimulus

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***Abstract*—Measuring the gain of an analog or mixed-signal block can be done by applying a sine wave signal or a rectangular signal at the input and measuring the amplitude of the block's output signal. Sometimes the drift of the DC offset of the block or of the tester's instruments is a challenge: if it is too high, it disturbs the measurement and reduces accuracy. In this case, using the binary Thue-Morse Sequence instead of a periodic rectangular or sine wave as an input reduces the effect of DC offset drift.**

Using the Thue-Morse Sequence also helps to reduce drift influence in a technique described in the literature. This technique allows for very accurate linearity measurements in ADCs, so that cheaper analog tester instruments or analog built-in self test can be used without losing accuracy.

I. Introduction

One test of the test suite for analog or mixed-signal IP blocks (like ADCs, DACs, amplifiers etc.; in the following part of this paper simply called IPs) involves the measurement of the amplification or gain. This also holds for ADCs and DACs, where the term 'gain' refers to the difference in analog input or output voltage per LSB of the digital signal.

The gain is usually measured by applying a rectangular or sine wave at the input and measuring the output amplitude. The gain of the IP is then calculated as the ratio of output amplitude to input amplitude.

Drift of the DC offset voltage sometimes disturbs these measurements. This offset voltage drift can be caused by many sources:

- drift in the IP's voltage reference,
- drift in the tester's instruments (or, if used, the analog Built-In Self Test (BIST) modules),
- low frequency (1/f) noise in MOSFETs,
- drift caused by temperature changes or fluctuations in the supply voltage etc.

Using a sufficiently high number of periods of the waveform usually allows for averaging the output amplitude, and thus reducing the effect of drift to an insignificant level. This method can be sufficient or even optimal if the time for the measurement is not critical. In IC production test, however, the available time is often so short that DC offset drift in the measurement path cannot be cancelled by averaging the output signal, and thus accuracy is reduced. In such cases, choosing the optimal input signal can help reduce sensitivity to drift.

In the following, we will show that the aperiodic Thue-Morse Sequence (TMS) as a stimulus waveform enables the complete cancellation of higher order drift terms – compared to a substantial, but not complete cancellation when using a periodic waveform.

The rest of this paper is organized as follows: Chapter II introduces the TMS. Chapter III describes a model for the measurement system and introduces the symbols used. Chapter IV provides a calculation for the IP's gain using the TMS. Chapter V generally compares the errors when using the TMS and the periodic rectangular signal. Chapter VI provides a calculation for these errors at the exponential decay function and at random noise. Chapter VII discusses some applications and Chapter VIII concludes.

II. The Thue-Morse Sequence

The Thue-Morse Sequence t(n) is a binary sequence that was first implicitly described by Prouhet in 1851. It was rediscovered by Thue in 1906, and later by Morse in 1921. It can be constructed as follows:

$t(0) = -1$
$t(2n) = t(n)$
$t(2n+1) = -t(n)$

The first terms of the TMS are:

$$t(n) = -1,\ 1,\ 1,\ -1,\ 1,\ -1,\ -1,\ 1,.... \qquad (1)$$

The TMS is an infinite, self-similar, recurrent, but aperiodic sequence. Due to some of its astonishing properties, it is used mainly in mathematics [1] and crystal physics [2], but also in such diverse fields as chess theory [1], counter synchronization [3], and computer-generated music [4].

Surprisingly, only one reference is known in metrology [5]: a paper that describes a way of measuring the linearity of high-precision ADCs in a low-precision environment. The authors intuitively constructed a binary sequence identical to the TMS, but since they were apparently unaware that they were using the TMS, they didn't provide a mathematical proof that their sequence provides better drift rejection than a periodic rectangular signal. The algorithm will be explained in more detail in Chapter VII B.

III. MODEL AND SYMBOLS USED

To illustrate the solution described in this paper, Figure 1 shows the IP under test (inner gray box) and the surrounding BIST shell or tester.

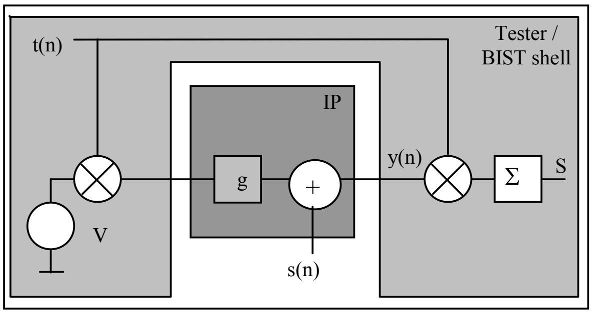

Figure 1: IP and measurement system with its input and output values

The constant voltage V is first multiplied (modulated) by the TMS t(n) and forms the IP's input. This signal is amplified with the IP's gain factor g (which is the value to be measured). An unknown perturbation (drift) s(n) is added – it can originate in the IP itself, or in the tester / BIST circuitry. The resulting IP output signal is called y(n). This output signal is then again multiplied (demodulated) with t(n). The resulting signal is summed up for all values n=0…N-1; the sum is called S. Note that the stimulus and the sampling of the IP's output are time-discrete.

The following symbols are used:
-V and V are constant stimuli levels
N is the number of samples
n is the index of the sample (n=0,1,2,…N-1)
t(n) is the n-th element of the Thue-Morse Sequence
g is the unknown IP amplification (gain)
s(n) is the perturbation due to DC drift
y(n) are the measured IP output values
S is the sum of the final values.

Of course the idea of noise-reduction by modulation and demodulation is far from new: it is used everywhere where the transmission path adds noise, e.g. in chopper-stabilized amplifiers, active sensors like strain gauges, various transmission systems etc. What is new in this paper, is the idea to use an aperiodic signal to obtain a complete cancellation of the lowest ordered noise or drift terms.

It is important that the setup described in Figure 1 is very simple – so it can be implemented in a BIST shell without a large area overhead.

IV. CALCULATION OF THE IP'S GAIN

For characterizing or testing the IP, we need to calculate the gain g with known y(n) and unknown perturbation s(n). As the perturbation s(n) is a finite, time-discrete signal, it can be expressed as a power series:

$$s(n) = \sum_{k=0}^{\infty} s_k n^k \qquad (n = 0 \ldots N-1) \tag{2}$$

The value S is defined as (cf. Figure 1):

$$S = \sum_{n=0}^{N-1} t(n) y(n) \tag{3}$$

As can be seen in Figure 1, y(n) is:

$$y(n) = g\ V\ t(n) + s(n) \tag{4}$$

Substituting (2) into (4), we obtain:

$$y(n) = g\ V\ t(n) + \sum_{k=0}^{\infty} s_k n^k. \tag{5}$$

Calculating S according to (3) using (5) results in:

$$S = \sum_{n=0}^{N-1} \left(g\ V\ t(n)\ \ t(n) + t(n) \sum_{k=0}^{\infty} s_k n^k \right) \tag{6}$$

This can be simplified, because the sum over the *g V t(n) t(n)* can be calculated separately. Because *t(n) t(n)* ≡ *1*, we get:

$$\sum_{n=0}^{N-1} t(n)\ \ t(n) gV = gVN \tag{7}$$

Thus (6) can then be written as:

$$S = gVN + \sum_{n=0}^{N-1} \left(t(n) \sum_{k=0}^{\infty} s_k n^k \right) \tag{8}$$

The TMS has an important and astonishing property [1], (it is because of this property that it is used in this paper): If N is a power of 2, then

$$\sum_{n=0}^{N-1} t(n)\ n^k = 0 \quad for \quad \forall\ \ k \le ld(N) \tag{9}$$

with ld(n) being the dual logarithm, i.e. ld(2)=1, ld(4)=2 etc. This causes the first ld(N)+1 terms s_k to disappear, thus (10) becomes:

$$S = gVN + \sum_{n=0}^{N-1} \left(t(n) \sum_{k=1+ld(N)}^{\infty} s_k n^k \right) \tag{10}$$

And, if only the perturbation terms s_k of order $k \le ld(N)$ are present, (10) becomes:

$$g = \frac{S}{N\ V} \tag{11}$$

Thus, we can calculate g when knowing S, even in the presence of the perturbation terms s_k with $k \le ld\ (N)$. Of these terms, the DC drift term linear with time is s_1 and the first derivative of the linear DC drift is s_2 – these two are the most important terms in a slowly drifting environment and are thus completely filtered out even with as few as N=8 samples.

V. Remaining Error of TMS and Rectangular Signal

To discuss the remaining error due to the use of the TMS, we firstly consider the periodic rectangular signal (-1, 1,-1, 1,-1, 1…), which is often used as a stimulus in IPs as described above. We then compute the error when using the TMS and the error when using the rectangular signal.

A. Rectangular signal

We define the rectangular signal a(n) that

$$a(n) = \begin{cases} -1 & n\ \ even \\ \ \ 1 & n\ \ odd \end{cases} \tag{12}$$

(cf. Figure 2). When applying it to an IP similar to Figure 1, the measured values y(n) are similar to (4):

$$y(n) = g\ V\ a(n) + \sum_{k=0}^{\infty} s_k n^k \tag{13}$$

In which S is similar to (8):

$$S = gVN + \sum_{n=0}^{N-1} \left(a(n) \sum_{k=0}^{\infty} s_k n^k \right) \tag{14}$$

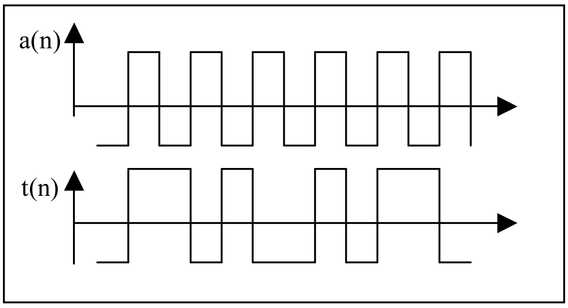

Figure 2: TMS and rectangular signal

B. Error when using the TMS and the rectangular signal

To compare the TMS and the rectangular signal, the error of S is interesting – i.e. how much does S deviate from the value it would have without perturbation, i.e. if $s(n) \equiv 0$:

This error is for the rectangular signal, from (14):

$$E_{RECT} = \sum_{n=0}^{N-1} \left(a(n) \sum_{k=0}^{\infty} s_k n^k \right) \tag{15}$$

And for the TMS, from (10):

$$E_{TMS} = \sum_{n=0}^{N-1} \left(t(n) \sum_{k=1+ld(N)}^{\infty} s_k n^k \right) \tag{16}$$

When knowing the statistics of the coefficients s_k, we are thus able to estimate and compare the errors when using the TMS or the rectangular signal.

VI. TMS and Rectangular Signal with Different Perturbation Signals

In the following, the two most frequent real-world perturbation (or drift) signals s(n) are used to compare a measurement with the TMS against one with the rectangular signal: the exponential decay function and noise, where we distinguish between white noise and low-pass filtered noise.

A. Exponential decay function

Often, the IP under measurement is not yet under fully settled conditions (IC temperature, supply voltage, reference voltage etc.) when the measurement starts. Test time constraints may prevent waiting until they have completely settled, or, in other cases, the measurement itself often unintentionally modifies conditions, such as temperature or supply voltage. The effect of this change in conditions is interpreted as the perturbation s(n). It often can be described with an exponential decay function:

$$s(n) = s(0)\exp(-n/T) \tag{17}$$

with T=Time constant. Exponential decay signals with T=5, T=10, and T=20 were used for the calculation (Figure 3).

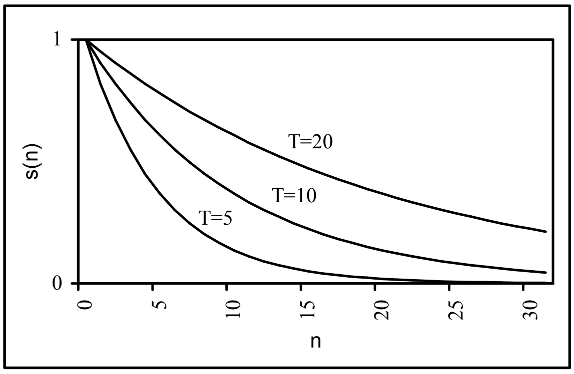

Figure 3: The exponential decay functions

Figure 4 shows the ratio of the remaining errors E_{TMS} / E_{RECT} (eq. (15) and (16)), if the perturbation function is an exponential decay function. Since a 2-sample TMS is the same as a 2-sample rectangular signal (namely –1, 1), also the errors for a rectangular wave and TMS are the same for N=2, so that the ratio is 1.

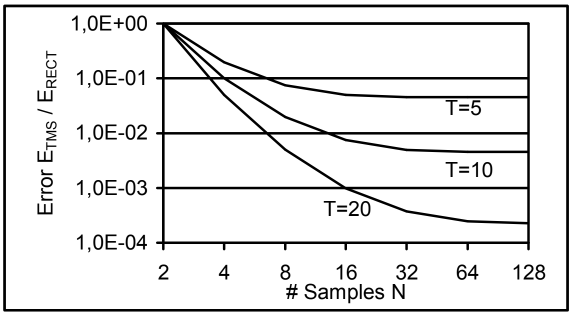

Figure 4: Remaining error for the exponential decay function: ratio of E_{TMS} / E_{RECT}

The impressive reduction of the error due to perturbation should not lead to losing sight of other sources of inaccuracy – of course an error reduction in the order of 10^{-3} will not be achievable in practice, because other sources of inaccuracy then will become dominant.

B. Noise

As opposed to the deterministic exponential decay function, noise is by its nature non-deterministic and can only be described with a certain spectrum in the frequency domain. Depending on what the physical root cause of the noise is, it is a white noise, 1/f or $1/f^2$ noise at its source. It can be low-pass filtered (e.g. noise caused by cross-sensitivity to thermal fluctuations; these fluctuations being filtered by thermally slow IC cases) and appear at the IP to be $1/f^2$ or even $1/f^3$ noise. The same holds for noise from voltage references: any noise on a voltage reference of an IP usually results in a corresponding noise at the IP's output. To reduce the effect of the voltage reference's noise, its voltage is often passed through a low-pass filter, and thus arrives at the IP as low-pass filtered noise.

For calculating the error when using the TMS or a rectangular signal, it is important to consider the spectrum of the noise. It is evident that the re-ordering of the stimulus sequence done by the TMS doesn't help with white noise, because in time-discrete systems, white noise has a value for each sample independent of the value of the previous sample. For the same reason, the periodic rectangular wave doesn't reduce the influence of white noise.

In low-pass filtered noise, however, both the TMS and the rectangular signal are efficient. This can be explained best for the TMS: in low-pass filtered noise, low frequencies dominate, so that the higher ordered perturbation terms s_k become smaller with rising order k. Thus, because the TMS eliminates the lower ordered s_k terms, it eliminates the biggest part of the perturbation.

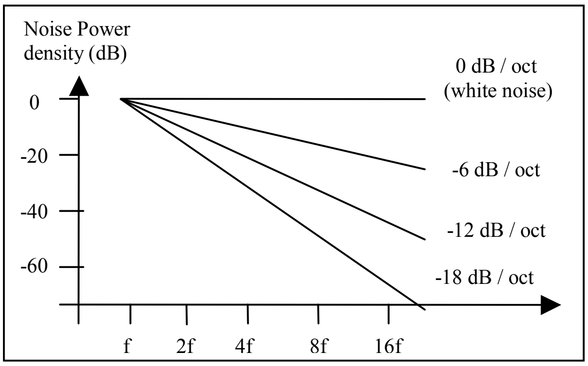

Figure 5: Noise with different falloff characteristics (arbitrary units)

The error due to noise was calculated numerically. It was found that 1/f noise (3 dB / octave falloff) is not reduced by the TMS. Also for $1/f^2$ noise (which is equivalent to a 6 dB / octave falloff), no improvement was found. In –12 dB / octave and –18 dB / octave noise, however, the improvement is considerable. Figure 6 shows the error reduction ratio (E_{TMS}/E_{RECT}) reduction in error for different sample numbers, and for –12 dB / octave and –18 dB / octave low pass filtered noise.

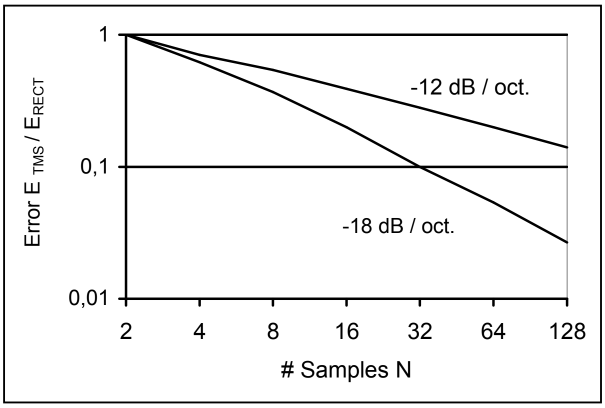

Figure 6: Remaining error for low-pass filtered noise: ratio of E_{TMS} / E_{RECT}

A natural question is, how does the suppression of low-ordered fluctuation terms in the time domain look like in the frequency domain? This question cannot be easily answered, because a direct relation between time and frequency domain only exists in linear time-invariant systems. The described modulation-demodulation scheme is linear, but, due to the aperiodic nature of the Thue-Morse Sequence, it is not time-invariant. This is also the reason why the influence on noise had to be simulated instead of being directly calculated.

Summarizing this chapter, using the TMS in the presence of DC drift that follows the exponential decay function, results in an impressive error reduction. In the presence of low-pass filtered noise, the error reduction is much less, but may in many applications still be significant. Since there is no improvement in white noise and in –6 dB / octave noise, there has to be a good low-pass pre-filtering of noise-generating signals, like band-gap voltages, to take advantage of the TMS.

VII. APPLICATIONS

While the proposed method addresses a quite wide area of applications in the field of testing and measurement, one has to remember that it is only beneficial if a number of conditions are met:

- The perturbation is mainly due to a slow DC offset drift, not to random noise. If white noise is dominant, however, the rectangular signal would be the better choice.
- The total number of samples is too small to average out the perturbation using the rectangular signal. This is usually due to restricted test time.
- The number of samples is (or can be made to be) a power of 2.
- The IP under measurement is linear and quasi-static, i.e. it does not remember the input history.

This is mostly the case in the following applications:

A. *Delta-Sigma DACs*

Delta-Sigma DACs usually consist of a one-bit converter followed by an analog reconstruction filter. These DACs are very linear, but usually have a high DC drift [6]. This makes them potential candidates for gain test with the proposed method. This applies especially to slow, high precision audio DACs.

B. *Linearity Test for ADCs*

While the proposed method directly addresses *gain* measurement, there is an interesting application for ADC *linearity* measurement [5]: The authors used a special algorithm they called SEIR (Stimulus Error Identification and Removal) [6]. For this, they applied to the ADC the sum of a slow periodic triangular signal and a faster TMS signal. The ADC thus received (nearly) the same triangular signal many times, some times with the overlayed TMS signal = –1, and some times with this TMS signal =1. The redundant information out of these two measurement sets was used to calculate the non-linearity of the input signal, and remove it from the ADC's output signal (Figure 7).

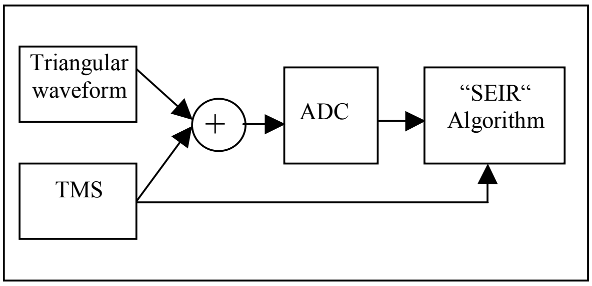

Figure 7: The setup for the linearity test

By doing so, the authors showed that a 16 bit ADC can be tested using a stimulus source with less than 7 bit linearity; the reference voltage showed a drift during the measurement of as much as 500 ppm. Even in this environment, the inaccuracy of the measurement – shown by simulation and by experiment – was much less than 1 LSB on 16-bit basis. In this setup, the use of the TMS overlay signal allowed for a perfect cancellation of the DC drift. Thus, as the authors also noted, there is a good chance to integrate a test signal generator with 7-bit accuracy within an analog BIST circuit for a 16 bit audio ADC.

This might help solve the permanent problem of analog BIST: on the one hand, the BIST modules are expected to be more precise than the IP they measure – on the other hand they have to be much smaller than the IP.

The authors of [5] didn't dive deeper into the properties of the TMS they used – as mentioned in chapter II, they were apparently unaware that they used an already well-known sequence with properties that perfectly suited their requirements.

Part of the motivation for this paper is to elucidate the mathematical foundation of that portion of their work that is related to the TMS. Combining the TMS-based gain measurement and the described TMS-based linearity measurement using the SEIR algorithm can be reasonably expected to facilitate analog testing.

C. Analog blocks

Amplifiers, and sensors that need an external excitation (optical sensors and sources, some micro electro-mechanical systems (MEMS), strain gauges, etc.) may be tested using the TMS, provided that the DC drift is an issue, the gain has to be measured with high precision, and a quasi-static condition can be achieved.

D. Other systems

The use of the TMS as a stimulus sequence is not restricted to production testing for analog / mixed signal IPs: in any system that needs to be stimulated in order to measure its transfer gain, it may be worth considering stimulating with the TMS.

Suppose, for example, that an engineer wants to know if probecard P_1 or probecard P_2 leads to a better contact yield, with contact yield gradually changing (drifting) from day to day – so that the s_1 linear drift term is present, in addition to an s_2 drift derivative, with the higher s_k terms being quite small. Each day one of the two probecards is used according to a scheme to be devised by the engineer. In this case it may be useful to test the probecard, not with the periodic rectangular-like scheme: P_1, P_2, P_1, P_2, P_1, P_2, P_1, P_2, but with the TMS-like scheme: P_1, P_2, P_2, P_1, P_2, P_1, P_1, P_2.

VIII. CONCLUSION

While the binary Thue-Morse Sequence has been used in many fields of mathematics and physics, it has apparently never been applied to the precise measurement of the IP sensitivity (or IP gain) in a fluctuating environment.

Whenever an IP needs a (binary) stimulus, it might be useful to stimulate, not with a rectangular (–1, 1, –1, 1, –1, 1…) signal, but with the said Thue-Morse Sequence. Then, for example, a sequence with 32 samples already completely removes the five lowest ordered perturbation terms, which, as we have shown, can lead to reducing the error due to DC drift by a factor of a hundred to a thousand.

However, the Thue-Morse Sequence as an excitation signal should only be used if the perturbation is a rather slowly changing DC drift, rather than white noise. Also the IP has to be linear and quasi-static at the input signal frequency used. Finally, reducing the perturbation by averaging using a periodic input signal is simpler, so the TMS should be primarily considered only if the available time is too short for averaging.

There is an – albeit quite small – number of applications in IC testing that fulfill the above requirements, mainly in delta-sigma DACs and amplifiers. Another interesting application is the linearity measurement of high-precision ADCs, which needs an additional algorithm and employs the TMS.

Both gain and linearity measurements with the described method can be done using a BIST shell with little area overhead.

The remaining error when using the Thue-Morse Sequence – compared to a periodic rectangular signal – is highly dependent on the particular application: from no difference if white noise dominates, to a huge improvement if white noise is negligible and only the linear DC drift and some higher ordered drift terms are present.

Changing the sequence of the input signal often requires very little effort; choosing the Thue-Morse Sequence can sometimes (but then, often to a high degree) minimize the disturbing factors, and thus increase accuracy and save (test) time and money.

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